Published by Institute of Physics Publishing for SISSA/ISAS
Received: *September 25, 2002*
Accepted: *October 04, 2002*

Supergravity duals of non-commutative wrapped D6-branes and supersymmetry without supersymmetry

Jan Brugués, Joaquim Gomis,[*] Toni Mateos and Toni Ramírez

Departament ECM, Facultat de Física, Universitat de Barcelona and
Institut de Física d'Altes Energies, Diagonal 647
E-08028 Barcelona, Spain
E-mail: jan@ecm.ub.es, gomis@ecm.ub.es, tonim@ecm.ub.es, tonir@ecm.ub.es

Abstract: We construct the supergravity solution in 11 dimensions describing D6-branes wrapped around a Kähler four-cycle with a B-field along the flat directions of the brane. The configuration is dual to an $\mathcal{N} = 2$ non-commutative gauge theory in $2 + 1$ dimensions. We also construct the four associated independent Killing spinors. The phenomenon of supersymmetry without supersymmetry appears naturally when compactifying to type-IIA or 8d gauged supergravity. Therefore, this solution also provides an 11d background with four supercharges and four-form flux, which is not obtainable from 8d gauged supergravity.

Keywords: D-branes, Brane Dynamics in Gauge Theories, AdS/CFT and dS/CFT Correspondence, Non-Commutative Geometry.

[*]Dipartimento di Fisica, Università di Firenze INFN, Sezione di Firenze, I-50125 Firenze

JHEP10(2002)016

Contents

1. Introduction

In the last two years, an extension of the AdS/CFT [1, 2, 3] correspondence to gauge field theories with less than maximal supersymmetry has been achieved via wrapped branes, and many supergravity duals have been constructed, see e.g. [4]–[30]. Gauged supergravities provided a useful tool to construct such configurations since they geometrically implement the twisting [31] of the field theories in a natural way. These supergravity solutions have been used to study qualitative aspects of non-perturbative gauge theories like quark confinement, chiral symmetry breaking, supersymmetry breaking and renormalization group flow.

On the other hand, supergravity duals of non-commutative (NC) theories with maximal supersymmetry have also been constructed [32]–[35] by switching on a background B-field. See [36] for a summary of all Dp-brane solutions.

The aim of this paper is to join both ideas and produce supergravity duals of non-maximally supersymmetric NC field theories by obtaining solutions of wrapped branes in a non-trivial B-field background. In particular, we will construct the sugra dual of an $\mathcal{N} = 2$ NC field theory in $2 + 1$ dimensions by wrapping D6-branes around a Kähler four-cycle.[1]

As we show, such solutions could not have been obtained using the corresponding 8d gauged supergravity [37], since even the unwrapped NC configuration looses all supersymmetry in the compactification. This is the well-known phenomenon of supersymmetry without supersymmetry [38], and it forces us to work directly in 11d sugra [39].

[1] We recall that in the presence of B-fields, the unwrapped D6-branes are expected to be free from the usual bulk gravity decoupling problems [34].

JHEP10(2002)016

In order to illustrate clearly this phenomenon, we first reconsider the unwrapped NC D6 in 11d which, unlike the commutative case, involves a non-trivial four-form flux. We give an ansatz, we derive and solve the first order BPS equations and, most importantly, we obtain the explicit form of the 16 Killing spinors. These allow us to interpret the solution as a non-threshold bound state of MKK-M5 (or D6-D4 in IIA), and to show which of its possible compactifications to type-IIA or 8d gauged sugra are supersymmetric.

We then apply the same method to the case of N D6 wrapping a calibrated Kähler four-cycle inside a Calabi-Yau three-fold. The 11d description (obtained in [17]) was purely gravitational and it was given by a 3d Minkowski spacetime times a Calabi-Yau four-fold. The metric was constructed in 8d sugra by demanding supersymmetry and then uplifted to 11d. We will construct the NC version of such configuration. The Killing spinors will allow us to interpret it as a non-threshold bound state of MKK-M5 (or D4-D6 in type IIA) with the D4 completely wrapped around the Kähler four-cycle. This background has 4 linearly realised supersymmetries and is dual (in the IR) to an $\mathcal{N}=2$ U(N) NC field theory in $2+1$ dimensions. Since the non commutativity is along the two spatial directions, such theory is non local in space, but free from unitarity or causality problems [40, 41].

The organization of the paper is as follows. In section 2 we reconsider the case of the unwrapped NC D6-branes and illustrate how supersymmetry without supersymmetry arises in compactifying. In section 3 we consider the case of the NC wrapped D6-branes, we obtain the new 11d supergravity solution and discuss its properties and its supersymmetry. In section 4 we give some conclusions.

2. Flat non-commutative D6-branes

2.1 Obtaining the solution in 11d

The purpose of this section is to find the supergravity solution describing N flat D6-branes in the background of a magnetic B-field. We will call this configuration a NC flat D6-brane.

The M-theory description [42, 43] of the commutative D6 is simply given by pure geometry, so we will work in 11d sugra. Nevertheless, since we will turn on a B-field in the IIA description, we will need to turn on a three-form to describe the NC case. We will first obtain the whole geometry and then take the near horizon limit.

The first step will be to make and ansatz for the bosonic fields (metric and $A_{[3]}$) of 11d sugra. In order to do so, recall that the 11d solution for N flat commutative D6-branes is the product of $\mathbb{R}^6$ with the euclidean Taub-Nut space [44]

$$ds^2_{(11)} = dx^2_{0,6} + H\left(dr^2 + r^2\left[d\theta^2 + \sin^2\theta d\phi^2\right]\right) + R^2 H^{-1}\left(d\psi + \cos\theta d\phi\right)^2\,, \tag{2.1}$$

with

$$H(r) = 1 + \frac{R}{r}\,, \qquad R = g_s N\sqrt{\alpha'}\,, \tag{2.2}$$

and N is the number of D6-branes.[2]

[2] The coordinates range is $0 \le \theta < \pi$, $0 \le \phi < 2\pi$ and $0 \le \psi < 4\pi/N$, the latter demanded by regularity of the metric about the origin.

JHEP10(2002)016

The Taub-Nut space is a U(1) bundle over $\mathbb{R}^3$ and it turns out to be a hyper-Kähler manifold with SU(2) holonomy. It is remarkable that the near horizon limit of this solution gives the product of $\mathbb{R}^6$ with an A_{N-1} ALE singularity, which is nothing but the orbifold $\mathbb{C}^2/Z_N$. Apart from the global identifications imposed when modding by Z_N, the metric is locally flat. Therefore, any analysis based only on local properties of the manifold will not be able to distinguish the actual manifold from flat space.

Now we would like to make a non-commutative deformation by turning on a B-field (in a IIA description) along the (x^5, x^6) plane. This will explicitely break the SO(1,6) isometry of the worldvolume to an SO(1,4) $\times$ SO(2), so that the ansatz for the metric is

$$\begin{aligned} ds^2_{(11)} &= \tau^2(r)\left[dx^2_{0,4} + \sigma^2(r)dx^2_{5,6} + H\left(dr^2 + r^2[d\theta^2 + \sin^2\theta d\phi^2]\right)\right] + \\ &+ \tau^{-4}(r)R^2H^{-1}\left(d\psi + \cos\theta d\phi\right)^2 . \end{aligned} \tag{2.3}$$

Note that the factor in front of the U(1) fiber is related to the one in front of the other ten coordinates through the ansatz for lifting IIA solutions to M-theory [45, 46]:

$$ds^2_{(11)} = e^{-2\Phi/3}ds^2_{\mathrm{IIA}} + e^{4\Phi/3}\left(dx^T + C_{[1]}\right)^2 , \tag{2.4}$$

where x^T is the M-theory coordinate. We also make an ansatz for the three-form that respects the U(1) monopole fibration

$$A_{[3]} = \chi(r)\, dx^5 \wedge dx^6 \wedge (d\psi + \cos\theta d\phi) . \tag{2.5}$$

We will determine the functions of our ansatz by demanding that the supersymmetry transformations admit a non-trivial Killing spinor. Since the background is bosonic, we just need to care about the gravitino variation

$$\delta\Psi_A = D_A\epsilon - \frac{1}{288}\left(\Gamma_A{}^{BCDE} - 8\delta_A{}^{[B}\Gamma^{CDE]}\right)F_{BCDE}\epsilon , \tag{2.6}$$

where $D_A = (\partial_A + \frac{1}{4}\omega_A{}^{\mathrm{CD}}\Gamma_{\mathrm{CD}})$ is the covariant derivative in flat coordinates and F_{BCDE} is the four field strength form.

In what follows it will be very important to make clear the vielbein basis that we are using, since the explicit form of the Killing spinors depends on it. We choose the following vielbein for the diagonal part of (2.3)

$$\begin{aligned} e^a &= \tau(r)\,dx^a , \qquad a = 0,\ldots,4 , \qquad e^i = \tau(r)\sigma(r)\,dx^i , \qquad i = 5,6 , \\ e^7 &= \tau(r)H^{1/2}(r)dr , \end{aligned} \tag{2.7}$$

while for the squashed S^3 we take

$$\begin{aligned} e^8 &= \tau(r)H^{1/2}(r)r\,\tilde{e}^1 , \qquad e^9 = \tau(r)H^{1/2}(r)r\,\tilde{e}^2 , \\ e^T &= \tau^{-2}(r)H^{-1/2}(r)R\,\tilde{e}^3 , \end{aligned} \tag{2.8}$$

with $\tilde{e}^i$ the typical vielbeins of a round S^3

$$\tilde{e}^1 = d\theta , \qquad \tilde{e}^2 = \sin\theta d\phi , \qquad \tilde{e}^3 = d\psi + \cos\theta d\phi . \tag{2.9}$$

Now we proceed to analyse the supersymmetry variations. Due to the SO(1,4) symmetry, the equations for $A = 0,1,2,3,4$ are equivalent. If we assume that the Killing spinors do not depend on the coordinates $\{x^0,\ldots,x^6\}$, these equations can be written as

$$(\cos\alpha\Gamma_{D6} + \sin\alpha\Gamma_{D4})\,\epsilon = -\epsilon\,, \tag{2.10}$$

with

$$\begin{aligned}\cos\alpha &= \frac{\chi'}{\chi}\tau^3 H R^{-1} r^2\,, \qquad & \sin\alpha &= -6\tau^3\chi^{-1}\tau'\sigma^2 H^{1/2} r^2\,,\\ \Gamma_{D6} &\equiv \Gamma_{0123456}\,, & \Gamma_{D4} &\equiv \Gamma_{01234T}\,.\end{aligned} \tag{2.11}$$

Since $\{\Gamma_{D6},\Gamma_{D4}\} = 0$, eq. (2.10) is telling us that we are obtaining a non-threshold bound state of D6-D4 from a IIA point of view, or a bound state MKK-M5 from an M-theory one [47]. To proceed, note that the eq. (2.10) can be rewritten as[3]

$$e^{-\alpha\Gamma_{56T}}\epsilon = -\Gamma_{D6}\epsilon\,, \tag{2.12}$$

whose most general solution is

$$\epsilon = e^{\frac{\alpha}{2}\Gamma_{56T}}\,\tilde{\epsilon}(r,\theta,\phi,\psi)\,, \qquad \text{with } \Gamma_{D6}\tilde{\epsilon}(r,\theta,\phi,\psi) = -\tilde{\epsilon}(r,\theta,\phi,\psi)\,. \tag{2.13}$$

Note that the angle α is a function of r. At this point we need to make an ansatz for $\tilde{\epsilon}$,

$$\tilde{\epsilon}(r,\theta,\phi) = f(r)e^{\frac{\theta}{2}\Gamma_{78}}e^{\frac{\phi}{2}\Gamma_{89}}\epsilon_0\,, \tag{2.14}$$

where ϵ_0 is a constant spinor verifying $\Gamma_{D6}\,\epsilon_0 = -\epsilon_0$.

One can plug our ansatz in the remaining supersymmetry variations and obtain the following set of first order, coupled, non-linear BPS equations

$$3\frac{\tau'}{\tau} + \frac{\sigma'}{\sigma} = 0\,, \tag{2.15}$$

$$\chi\chi' - 6R^2H^{-1}\sigma^4\frac{\tau'}{\tau} = 0\,, \tag{2.16}$$

$$\frac{3\tau'}{\tau} + \frac{\chi'}{2\chi} + \frac{H'}{2H} = 0\,. \tag{2.17}$$

The general solution depends on a total of three arbitrary constants. Two of them can be fixed by demanding that the solution reduces to the commutative one (2.1) when the $A_{[3]}$ is set to zero (commutative limit). The remaining arbitrary constant has a physical meaning: it is the strength of the non commutativity, that we call Θ. We have

$$\tau(r) = h^{1/6}\,, \qquad \sigma(r) = h^{-1/2}\,, \qquad \chi(r) = -\frac{\Theta R}{Hh}\,, \qquad f(r) = h^{1/12}(r)\,, \tag{2.18}$$

where we have defined

$$h(r) = 1 + \Theta^2 H^{-1}\,. \tag{2.19}$$

[3] For a different configuration of M branes an analogous technique was used in [48].

Summarizing, the 11d metric, three-form and the Killing spinors are given by

$$
\begin{aligned}
ds^2_{(11)} &= h^{1/3}\left(-dx^2_{0,4} + h^{-1}dx^2_{5,6} + H\left[dr^2 + r^2 d\Omega_2^2\right]\right) + \\
&\quad + H^{-1}h^{-2/3}R\left(d\psi + \cos\theta d\phi\right)^2 , \qquad (2.20) \\
A_{[3]} &= -\frac{\Theta R}{Hh}\, dx^5 \wedge dx^6 \wedge (d\psi + \cos\theta d\phi) , \\
\epsilon(r,\theta,\phi,\psi) &= h^{1/12}(r) e^{\frac{\alpha(r)}{2}\Gamma_{56T}} e^{\frac{\theta}{2}\Gamma_{78}} e^{\frac{\phi}{2}\Gamma_{89}} \epsilon_0 \qquad (2.21)
\end{aligned}
$$

with

$$
\Gamma_{D6}\epsilon_0 = -\epsilon_0 , \qquad \cos\alpha = h^{-1/2} , \qquad \sin\alpha = \Theta(Hh)^{-1/2} . \qquad (2.22)
$$

This solution describes the whole geometry of N flat NC D6-branes and the number of independent Killing spinors is 16. The configuration corresponds to a bound state of N MKK monopoles and N M5 branes, or a bound state of N D6-D4 branes in type IIA. If we want to use this background, in the spirit of the AdS/CFT [1, 2, 3] correspondence, to study the dual NC field theory, we must take the near horizon limit, which consists of taking $\alpha' \to 0$ keeping fixed

$$
u = \frac{r}{\alpha'} , \qquad \tilde{\Theta} = \alpha'\Theta , \qquad g^2_{\rm YM} = g(\alpha')^{3/2} . \qquad (2.23)
$$

After a suitable change of variables,[4] the metric and the three-form become [36]

$$
ds^2_{11} = h^{1/3}\left[dx^2_{0,4} + h^{-1}dx^2_{5,6} + dy^2 + \frac{y^2}{4}\left(d\Omega^2_{(2)} + h^{-1}[d\psi + \cos\theta d\phi]^2\right)\right] , \qquad (2.24)
$$

$$
A_{[3]} = -\frac{\tilde{\Theta}}{4Ng^2_{\rm YM}}\frac{y^2}{h} dx^5 \wedge dx^6 \wedge (d\psi + \cos\theta d\phi) , \qquad h(y) = 1 + \left(\frac{\tilde{\Theta} y}{2Ng^2_{\rm YM}}\right)^2 . \qquad (2.25)
$$

The number of supersymmetries continues to be 16 so, unlike the commutative case, there is no enhancement. Note that this background should be in the IR the large-N dual of a $6+1$ non-commutative gauge theory with 16 supercharges.

Finally, we would like to consider the commutative limit of (2.20) and (2.24). Sending $\Theta \to 0$ implies $h \to 1$. In such limit, eq. (2.20) collapses to eq. (2.1) and the 16 spinors become simply

$$
\epsilon(\theta,\phi) = e^{\frac{\theta}{2}\Gamma_{78}} e^{\frac{\phi}{2}\Gamma_{89}} \epsilon_0 , \text{ with } \Gamma_{D6}\epsilon_0 = -\epsilon_0 . \qquad (2.26)
$$

On the other hand, in the commutative limit, eq. (2.24) becomes the aforementioned A_{N-1} singularity. Apart from the previous 16 spinors, it also admits the following 16 ones

$$
\epsilon(\psi) = e^{-\frac{\psi}{2}\Gamma_{89}} \epsilon_0 , \text{ with } \Gamma_{D6}\epsilon_0 = \epsilon_0 . \qquad (2.27)
$$

Note that they have a different eigenvalue with respect to Γ_{D6}. Modding out by the Z_N global identifications brings the number of supersymmetries back to 16. Only for N = 1, flat space, we have a true enhancement of susy.

[4] Explicitely $u = y^2/4Ng_{\rm YM}$.

2.2 Compactification to type IIA and supersymmetry

As is well known, any configuration of 11d supergravity can be consistently reduced to 10d IIA supergravity as long as it is a U(1) fibration over a ten-dimensional base space [49]. The bosonic part of the reduction ansatz is [45, 46]

$$
\begin{aligned}
ds^2_{(11)} &= e^{-2\phi/3} ds^2_{\mathrm{IIA}} + e^{4\phi/3}\left(dx^T + C_{[1]}\right)^2 , && (2.28)\\
A_{[3]} &= -C_{[3]} + dx^T \wedge B_{[2]} , && (2.29)
\end{aligned}
$$

where ∂_{x^T} is the Killing vector that generates the U(1) isometry.

Indeed, the ansatz does not finish here. First of all, the vielbeins must be selected so that the first ten of them do not depend on x^T.[5] Second, once we fix the vielbein, the supersymmetry parameter must also be independent of x^T. These comments are relevant, since we will show that some of our configurations fit perfectly into the ansatz (2.28) but do not verify the condition on the susy parameter. In such cases, one produces solutions of the IIA equations of motion, but they do not typically have any linearly realised supersymmetry [51]. Note as well, that all the metrics in this section have at least two different U(1) isometries, generated by the Killing vectors ∂_ψ and ∂_ϕ. The amount of supersymmetry preserved in the reduction to IIA-sugra, will depend on whether we have non commutativity or not.

I. Non-commutative case: we will work with the basis (2.8). Since it does not explicitely depend on ϕ nor ψ, it can be used for the reduction along both U(1) isometries. Fixed the vielbeins, we see that, in both the whole geometry (2.20) and in the near horizon (2.24), the sixteen spinors depend on ϕ but not on ψ. As a consequence, reducing along the ∂_ψ isometry will preserve the whole 16 supercharges, and we obtain the same supersymmetric configuration found in e.g. [36].

If we want to reduce along ∂_ϕ, we will produce a solution of type-IIA supergravity which will not be supersymmetric. In order to do so, we need to reexpress our metric (2.20) in a form that makes the new U(1) fibration more explicit

$$
\begin{aligned}
ds^2_{(11)} = \ & h^{1/3}\left(dx^2_{0,4} + h^{-1}dx^2_{5,6} + H\left[dr^2 + r^2 d\theta^2\right]\right) + \\
& + h^{-1/3}\lambda^{-1} R r^2 \sin^2\theta\, d\psi^2 + \lambda\left(d\phi + A_{[1]}\right)^2
\end{aligned} \qquad (2.30)
$$

with

$$
A_{[1]} \equiv \lambda^{-1} H^{-1} h^{-2/3} R \cos\theta\, d\psi , \qquad \lambda(r,\theta) \equiv H h^{1/3} r^2 \sin^2\theta + H^{-1} h^{-2/3} R \cos^2\theta . \quad (2.31)
$$

So reducing along ∂_ϕ gives

$$
\begin{aligned}
ds^2_{\mathrm{IIA}} = \ & \lambda^{1/2} h^{1/3}\left(dx^2_{0,4} + h^{-1}dx^2_{5,6} + H[dr^2 + r^2 d\theta^2]\right) + \\
& + \lambda^{-1/2} h^{-1/3} R r^2 \sin^2\theta\, d\psi^2 ,
\end{aligned} \qquad (2.32)
$$

[5] This statement can be made more rigorous by computing the more intrinsic Lie-Lorentz derivative [50] with respect to the Killing vectors. For the cases considered in this paper, such derivative collapses to the usual one.

$$e^{4\Phi/3} = \lambda(r,\theta)\,, \qquad B_{[2]} = -\frac{\Theta R}{Hh}\cos\theta\, dx^5\wedge dx^6\,, \tag{2.33}$$

$$C_{[1]} = A_{[1]}\,, \qquad C_{[3]} = \frac{\Theta R}{Hh}\, dx^5\wedge dx^6\wedge d\psi\,. \tag{2.34}$$

It can be checked that such configuration is a solution of the type-IIA equations of motion and that it does not preserve any supercharge. This is an example of the phenomenon of supersymmetry without supersymmetry [38].

II. Commutative case: we can obtain the commutative configurations just by letting $\Theta\to 0$ in the former equations. When reducing along ψ, this just gives the usual D6-brane metrics of type IIA. Nevertheless, when reducing along ϕ we have to distinguish whether we are or not in the near horizon limit. If we are still in the whole geometry, the only Killing spinors are those in (2.26), which explicitely depend on ϕ. So the ∂_ϕ-reduction kills all supersymmetry, and it simply gives (2.32) with the replacement $h\to 1$.

The major change comes in the near horizon limit of the commutative case which, recall, it is just locally flat space in 11d. As we said, there is a local enhancement to 32 supersymmetries, and sixteen new spinors appear. They are the ones of (2.27), and they depend only on ψ. Therefore, they all survive the ∂_ϕ-reduction and must produce locally a 16-supersymmetric configuration. By construction, this solution is nothing but the near horizon limit of (2.32) with $h=1$. The expected 16 Killing spinors can simply be obtained using the reduction formulas [45, 46]

$$\epsilon(\psi,\theta) = e^{\Phi/6}e^{-\frac{\psi}{2}\Gamma_{89}}\epsilon_0\,, \text{ with } \Gamma_{D6}\epsilon_0 = \epsilon_0\,. \tag{2.35}$$

2.3 Compactification to 8d gauged sugra and supersymmetry

Gauged supergravities have recently been exploited to obtain configurations of wrapped branes, since they provide a natural method to implement geometrically the twistings of the supersymmetric field theories [31]. In this subsection we show that one runs into trouble when trying to use them to obtain NC duals via wrapped branes. In particular, we will show that the NC flat configurations, both for the whole geometry (2.20) and for the near-horizon limit (2.24), are not supersymmetric from the point of view of 8d gauged sugra. The compactification of M-theory on an SU(2) manifold was worked out in [37], and it gave a maximal 8d SU(2) gauged supergravity. This theory was used in [10, 14, 17, 26, 27] to obtain sugra duals of non-maximally supersymmetric field theories. Now we would like to see if our configuration (2.20) and (2.24) could have been found by using this 8d gauged sugra. To answer this question, the first thing to do is to choose the vielbein that is implicit in [37]. In particular one needs to work with the SU(2) left invariant one-forms for the squashed S^3 part of the metric. So that, instead of (2.8), one should use

$$\hat e^8 = \tau(r)H^{1/2}(r)rw^1\,, \qquad \hat e^9 = \tau(r)H^{1/2}(r)rw^2\,, \qquad \hat e^T = \tau^{-2}(r)H^{-1/2}(r)Rw^3\,, \tag{2.36}$$

with[6]

$$\begin{aligned} w_1 &= -\cos\psi d\theta - \sin\theta\sin\psi d\phi\,, \qquad w_2 = -\sin\psi d\theta + \sin\theta\cos\psi d\phi\,, \\ w_3 &= -d\psi - \cos\theta d\phi\,. \end{aligned} \tag{2.37}$$

[6]Note that the signs have been chosen so that both basis share the same orientation.

We will call (2.36) the w-base and (2.8) the e-base. It is easy to work out the form of the spinor in this new base, since we have just performed a local lorentz transformation. Explicitely, it can be seen that the w-base can be obtained from the e-base by performing a rotation of π along x^9, followed by a rotation of angle $-\psi$ along x^T. So the Killing spinors will transform with the (inverse) spin 1/2 representation of such rotations

$$\epsilon' = e^{-\psi \frac{\Gamma_{89}}{2}} e^{\pi \frac{\Gamma_{T8}}{2}} \epsilon = \Gamma_{T8} e^{\psi \frac{\Gamma_{89}}{2}} \epsilon \,. \tag{2.38}$$

We can now see that all the Killing spinors obtained in 11d, after the change of base, become $\{\theta, \phi, \psi\}$-dependent, while only the 16 ones that produced the enhancement in the commutative near-horizon limit (2.27) become constant spinors.

The compactification on a group manifold [52] assumes that supersymmetry parameters do not depend on the internal space coordinates. So we can already predict that only the commutative near-horizon limit will appear to be supersymmetric in 8d. Indeed, this made possible the obtention of a stack of N parallel flat D6-branes by analysing the BPS equations of 8d [55]. On the contrary, the NC configurations could never have been obtained by making an ansatz in 8d and looking at the susy variations. As an example, the configuration (2.24) can be reduced to 8d since it fits into the bosonic ansatz [37], and produces

$$ds^2_{(8)} = \frac{g}{4} y \, h^{1/3} \left(dx^2_{0,4} + h^{-1} dx^2_{5,6} + dy^2 \right) , \tag{2.39}$$

$$e^{2\phi/3} = \frac{g}{4} y \,, \qquad e^{\lambda} = h^{1/6} \,, \tag{2.40}$$

$$G_{[2]} = -\frac{\Theta g^2}{16 N g^2_{\rm YM}} \frac{y^2}{h} \, dx^5 \wedge dx^6 \,, \qquad G_{[3]} = -\frac{\Theta g}{4 N g^2_{\rm YM}} \frac{y}{h^2} \, dx^5 \wedge dx^6 \wedge dy \,, \tag{2.41}$$

where λ is a scalar field on the coset space $\mathrm{SL}(3,\mathbb{R})/\mathrm{SO}(3)$ and $G_{[2]}$ and $G_{[3]}$ are field strength forms of 8d sugra. This is again a solution of the 8d equations of motion but, as can be explicitely seen, it is not supersymmetric. In the next section we will obtain the wrapped version of all these configurations, and we will apply the same arguments to proof that the NC cases could not have been found from 8d sugra.

3. Non-commutative wrapped D6-branes

The configuration of N D6-branes wrapping a Kähler four-cycle inside a Calabi-Yau threefold was obtained in [17]. In this section we will first discuss some issues of its supersymmetry properties and then we will obtain its NC deformation.

3.1 Commutative wrapped D6-branes

By using 8d gauged supergravity, the purely gravitational 11d description of such configurations was obtained in [17]

$$ds^2_{(11)} = dx^2_{0,2} + \frac{3}{2}(r^2 + l^2) ds^2_{\rm cycle} + U^{-1} dr^2 + \frac{r^2}{4} \left(d\theta^2 + \sin^2\theta d\phi^2 \right) + \frac{1}{4} U r^2 \sigma^2 \,, \tag{3.1}$$

where σ described the following U(1) fibration

$$\sigma = d\psi + \cos\theta d\phi + A_{[1]}\,, \text{ with } dA_{[1]} = 6J_{[2]}\,. \tag{3.2}$$

Here, $J_{[2]}$ is the Kähler form of the four-cycle chosen and $\mathrm{U}(r) = \frac{3r^4+8l^2r^2+6l^4}{6(r^2+l^2)^2}$. This solution has the topology of $\mathbb{R}^{1,2}\times \mathrm{CY}_4$, the Calabi-Yau four-fold being a $\mathbb{C}^2$ bundle over the Kähler four-cycle $\mathcal{K}_4$.[7,8] Alternatively, it can be seen as a cone with $r = cte$ hypersurfaces described by a U(1) bundle over the base $S^2\times\mathcal{K}_4$.[9] In the following we choose the vielbeins so that the Kähler form of the such base is written as $J_{[2]} = e^3\wedge e^6 + e^4\wedge e^5 + e^8\wedge e^9$.

Performing the same analysis of the supersymmetry variations again, one finds the following Killing spinors

$$\epsilon(\psi) = e^{-\frac{\psi}{2}\Gamma_{89}}\epsilon_0\,, \tag{3.3}$$

with ϵ_0 a constant spinor subject to

$$\Gamma_{D6}\,\epsilon_0 = \epsilon_0\,, \qquad \Gamma_{36}\,\epsilon_0 = \Gamma_{45}\,\epsilon_0 = \Gamma_{89}\,\epsilon_0\,. \tag{3.4}$$

The first condition just signals the presence of the D6, while the other two are the usual projections of a Calabi-Yau three-fold. Altogether, the configuration preserves only 1/8 of the 32 supersymmetries.

In [17] the compactification to IIA was performed along the immediate U(1) isometry generated by ∂_ψ. This lead to the following type-IIA configuration[10]

$$ds^2_{\mathrm{IIA}} = e^{2\Phi/3}\left[dx^2_{0,2} + \frac{3}{2}(r^2+l^2)ds^2_{\mathrm{cycle}} + U^{-1}dr^2 + \frac{r^2}{4}\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right], \tag{3.5}$$

$$e^{4\Phi/3} = \mathrm{U}(r)r^2\,, \qquad C_{[1]} = \frac{1}{2}\left(\cos\theta d\phi + A_{[1]}\right). \tag{3.6}$$

A probe brane analysis showed that there was no moduli space. This can now be understood, since the Killing spinors (3.3) depend on the compactification coordinate and, therefore, do not fit in the reduction ansatz. Thus, as can be checked explicitely, (3.5) is a solution of the type-IIA sugra equations of motion, although it is not supersymmetric at all. This is another example of supersymmetry without supersymmetry, as discussed in section 2.2. Furthermore, this reduction did not produce the mentioned configuration of D6-branes inside a Calabi-Yau. As can be seen directly from the metric (3.5), the six-dimensional space spanned by the four-cycle and (θ,ϕ) has the structure of a direct product $\mathcal{K}_4\times S^2$ instead of being a fibration over the four-cycle. It cannot therefore be the claimed Calabi-Yau three-fold.

[7] The metric for this Calabi-Yau four-fold was first found in [53] in a completely different approach.

[8] This construction exemplifies the uplift from a manifold with SU(3) holonomy in type IIA to a manifold with SU(4) holonomy in M Theory [54].

[9] For simplicity, in this paper we will only consider the four-cycle $\mathcal{K}_4 = S^2\times S^2$, although the results can be generalised to any other choice. So, in this case, $ds^2_{\mathrm{cycle}} = 1/6\left(d\theta_1^2 + \sin^2\theta_1\,d\phi_1^2 + d\theta_2^2 + \sin^2\theta_2\,d\phi_2^2\right)$ and $A_{[1]} = \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2$.

[10] There is a typo in [17, eq (28)].

To avoid such phenomenon, one can try to reduce along other U(1) isometries, like the one generated by the Killing vector ∂_ϕ. In order to do so, the metric (3.1) must be rewritten in a form that makes explicit the new U(1) fibration, i.e.

$$\begin{aligned} ds^2_{(11)} &= dx^2_{0,2} + \frac{3}{2}(r^2+l^2)ds^2_{\text{cycle}} + U^{-1}dr^2 + \frac{r^2}{4}\left(d\theta^2 + mB^2_{[1]}\right) + \\ &\quad + \tilde{H}^{-1}(r,\theta)\left[d\phi - Uf^{-1}\cos\theta B_{[1]}\right]^2 , \end{aligned} \tag{3.7}$$

with the definitions

$$f(\theta,r) \equiv \sin^2\theta + \mathrm{U}(r)\cos^2\theta\,, \qquad m(\theta,r) \equiv \left(U^{-1} + \cot g^2\theta\right)^{-1}, \tag{3.8}$$

$$B_{[1]} \equiv d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2\,, \qquad \tilde{H}(r,\theta) \equiv \frac{4}{f(r,\theta)r^2}\,. \tag{3.9}$$

Now both the metric and the supersymmetry parameters verify the ansatz required to reduce down to IIA along ϕ, and it yields

$$ds^2_{\text{IIA}} = e^{2\Phi/3}\left(dx^2_{0,2} + \frac{3}{2}(r^2+l^2)ds^2_{\text{cycle}} + U^{-1}dr^2 + \frac{r^2}{4}\left[d\theta^2 + mB^2_{[1]}\right]\right), \tag{3.10}$$

$$e^{4\Phi/3} = \frac{r^2}{4}f(r,\theta)\,, \qquad C_{[1]} = -Uf^{-1}\cos\theta B_{[1]}\,. \tag{3.11}$$

Note that all these background fields depend now on both the radial coordinate r and the angle θ. It can be checked that this configuration is a solution of the equations of motion and that it has the expected four supersymmetries.

3.2 Non-commutative wrapped D6

Just as we did in section 2, now we will look for a NC deformation of the wrapped D6-branes, (3.7), by turning on a B-field, in this case, along the (x_1,x_2) plane. As before, we explicitely break the worldvolume SO(1,2) symmetry to $R\times$ SO(2). As in the unwrapped case, we will also make use of the fact that, in 11d, the factors in front of the 10d part of the metric and in front of the U(1) fiber are related through the lifting ansatz (2.4). Therefore, our ansatz for the bosonic fields is[11]

$$\begin{aligned} ds^2_{(11)} &= \tau^2(r,\theta)\left[-dx^2_0 + \sigma^2(r,\theta)dx^2_{1,2} + \frac{3}{2}(r^2+l^2)ds^2_{\text{cycle}} + U^{-1}dr^2 + \frac{r^2}{4}\left(d\theta^2 + mB^2_{[1]}\right)\right] + \\ &\quad + \tau^{-4}(r,\theta)\,\tilde{H}^{-1}\left[d\phi - Uf^{-1}\cos\theta B_{[1]}\right]^2 , \end{aligned} \tag{3.12}$$

$$A_{[3]} = \chi(r,\theta)\; dx^1\wedge dx^2\wedge\left[d\phi - Uf^{-1}\cos\theta B_{[1]}\right]. \tag{3.13}$$

Note that we allow the functions of the ansatz to depend on θ. Now we proceed to make an ansatz for the spinor. Just like in the NC flat case, we expect that we will obtain a projection signaling a bound state of MKK-M5

$$\left(\cos\alpha\Gamma_{D6} + \sin\alpha\tilde{\Gamma}_{D4}\right)\epsilon = \epsilon\,, \tag{3.14}$$

[11] We use the definitions of (3.8) for the functions $f(r,\theta)$ and $m(r,\theta)$.

for some angle $\alpha(r,\theta)$ to be determined. Notice that since now the B-field will be along (x^1,x^2), we expect the D4 to span the directions $\{x^0x^3x^4x^5x^6\}$, so that $\tilde\Gamma_{D4}=\Gamma_{03456T}$. As in the unwrapped case, see (2.10), (2.13), eq. (3.14) implies

$$\epsilon(r,\theta,\phi,\psi)=e^{\frac{\alpha(r,\theta)}{2}\Gamma_{12T}}\tilde\epsilon(r,\theta,\phi,\psi)\,,\ \text{with}\ \Gamma_{D6}\,\tilde\epsilon=\tilde\epsilon\,. \tag{3.15}$$

Now we are ready to obtain the BPS equations by imposing that the supersymmetry variation of the gravitino vanishes (2.6). The most immediate relations come from making them compatible for $A=0$ and $A=1,2$ and give[12]

$$3\,\frac{\tau'}{\tau}+\frac{\sigma'}{\sigma}=0\,,\qquad 3\frac{\dot\tau}{\tau}+\frac{\dot\sigma}{\sigma}=0\,, \tag{3.16}$$

whose integration yields $\sigma=\tau^{-3}$. The $A=5,6$ equations imply that

$$\left(\partial_\psi+\frac{\Gamma_{36}}{2}\right)\epsilon=0\,,\qquad \text{and}\qquad \Gamma_{36}\,\epsilon=\Gamma_{45}\,\epsilon\,. \tag{3.17}$$

while the $A=7$ equation implies that

$$\tau^{-6}+\tilde H\tau^6\chi^2=1\,. \tag{3.18}$$

Now taking a linear combination of the $A=1,3,9$ equations, and assuming that the spinor does not depend on the fiber coordinate ϕ, one reaches another constraint similar to (2.10)

$$(\cos\beta\Gamma_{3689}+\sin\beta\Gamma_{3679})\,\epsilon=-\epsilon\,, \tag{3.19}$$

with

$$\cos\beta=U^{1/2}f^{-1/2}\cos\theta\,,\qquad \sin\beta=f^{-1/2}\sin\theta\,. \tag{3.20}$$

Since the matrices Γ_{3689} and Γ_{3679} anticommute, we can proceed as in (2.13), and rewrite this equation as

$$e^{-\beta\Gamma_{78}}\epsilon=-\Gamma_{3689}\epsilon\,, \tag{3.21}$$

whose most general solution is

$$\epsilon(r,\theta,\psi)=e^{\frac{\alpha(r,\theta)}{2}\Gamma_{12T}}e^{\frac{\beta(r,\theta)}{2}\Gamma_{78}}\tilde{\tilde\epsilon}(r,\theta,\psi)\,,\qquad \text{with}\qquad \Gamma_{D6}\tilde{\tilde\epsilon}=-\Gamma_{3689}\tilde{\tilde\epsilon}=\tilde{\tilde\epsilon}\,. \tag{3.22}$$

Plugging this into (3.17) allows us to write down the final ansatz for the spinor:

$$\tilde{\tilde\epsilon}(r,\theta,\psi)=\gamma(r,\theta)e^{-\frac{\psi}{2}\Gamma_{89}}\epsilon_0\,,\ \text{with}\ \Gamma_{D6}\epsilon_0=-\Gamma_{3689}\,\epsilon_0=\epsilon_0\,. \tag{3.23}$$

The first order BPS equations are

$$6\frac{\tau'}{\tau}+\frac{\chi'}{\chi}+\frac{\tilde H'}{\tilde H}=0\,, \tag{3.24}$$

$$\dot\alpha-\frac{1}{2}\tilde H^{1/2}\tau^6\dot\chi=0\,,\qquad \alpha'-\frac{1}{2}\tilde H^{1/2}\tau^6\chi'=0\,, \tag{3.25}$$

$$\frac{\gamma'}{\gamma}-\frac{\tau'}{2\tau}=0\,,\qquad \frac{\dot\gamma}{\gamma}-\frac{\dot\tau}{2\tau}=0\,. \tag{3.26}$$

[12]We use primes for ∂_r and dots for ∂_θ. Also, the integration constant is set to one in order to recover the commutative case when the three-form vanishes.

They can be solved analytically and, after fixing the integration constants to reproduce the commutative case when $A_{[3]}$ vanishes, one obtains

$$\begin{aligned} \tau = \tilde{h}^{1/6}\,, \qquad \chi = -\frac{\Theta}{\tilde{H}\tilde{h}}\,, \qquad \gamma = \tilde{h}^{1/12}\,, \\ \cos\alpha = -\tilde{h}^{-1/2}\,, \qquad \sin\alpha = -\Theta(\tilde{H}\tilde{h})^{-1/2}\,, \end{aligned} \tag{3.27}$$

with

$$\tilde{h}(r,\theta) = 1 + \Theta^2 \tilde{H}^{-1}(r,\theta)\,. \tag{3.28}$$

So the whole solution for the metric, three-form and Killing spinor is

$$\begin{aligned} ds^2_{(11)} &= \tilde{h}^{1/3}\left(-dx_0^2 + \tilde{h}^{-1}dx_{1,2}^2 + \frac{3}{2}(r^2+l^2)ds^2_{\text{cycle}} + U^{-1}dr^2 + \frac{r^2}{4}\left[d\theta^2 + mB_{[1]}^2\right]\right) + \\ &\quad + \tilde{h}^{-2/3}\tilde{H}^{-1}\left(d\phi - Uf^{-1}\cos\theta B_{[1]}\right)^2\,, \qquad (3.29) \\ A_{[3]} &= -\frac{\Theta}{\tilde{H}\tilde{h}}\,dx^1\wedge dx^2 \wedge \left(d\phi - Uf^{-1}\cos\theta B_{[1]}\right)\,, \\ \epsilon(r,\theta,\psi) &= \tilde{h}^{1/12}(r,\theta)e^{\frac{\alpha(r,\theta)}{2}\Gamma_{12T}}e^{\frac{\beta(r,\theta)}{2}\Gamma_{78}}e^{-\frac{\psi}{2}\Gamma_{89}}\epsilon_0 \qquad (3.30) \end{aligned}$$

with the constant spinor ϵ_0 subject to the following independent constraints

$$\Gamma_{D6}\,\epsilon_0 = \epsilon_0\,, \qquad \Gamma_{36}\,\epsilon_0 = \Gamma_{45}\,\epsilon_0 = \Gamma_{89}\,\epsilon_0\,. \tag{3.31}$$

Note that the introduction of the B-field has not broken any extra supersymmetry, and the configuration still preserves 4 real supercharges. The metric (3.29) and the three-form (3.30) should be the supergravity dual of a $2+1$ $\mathcal{N}=2\,\mathrm{U(N)}$ field theory with non commutativity along the (x^1,x^2) plane. As in [17], the field content should consist of a vector multiplet.

Note also that this solution provides an example of M-theory compactification with fluxes. The topology is $\mathbb{R}^3\times\mathbb{X}_8$, with $\mathbb{X}_8$ the non-Ricci-flat internal manifold. $\mathbb{X}_8$ consists of a complicated four dimensional fibration over the Kähler base space $S^2\times S^2$. Remarkably, we can smoothly send to zero the non commutativity, so that the $A_{[3]}$ flux goes to zero and $\mathbb{X}_8$ becomes an SU(4)-holonomy Calabi-Yau four-fold. From a IIA perspective it describes a non-threshold bound state of D6-D4 branes with the $D4$ wrapped around the four-cycle, so that the arrays are

IIA	x^0	x^1	x^2	θ_1	θ_2	ϕ_2	ϕ_1	r	θ	ψ
D6	–	–	–	–	–	–	–			
D4	–			–	–	–	–			

11d	x^0	x^1	x^2	θ_1	θ_2	ϕ_2	ϕ_1	r	θ	ψ	ϕ
MKK	–	–	–	–	–	–	–				
M5	–			–	–	–	–				–

3.3 Compactifications to type-IIA and to 8d gauged sugra

In this subsection we will apply to the NC wrapped configuration (3.29) the same arguments of section 2.2 in order to discuss which compactifications preserve supersymmetry.

Since the susy parameters (3.30) depend only (θ, ψ), a U(1) reduction along ϕ will produce a type-IIA solution preserving the four supercharges. Explicitely

$$\begin{aligned} ds^2_{\rm IIA} &= e^{2\Phi/3}\tilde{h}^{1/3}\Big(- dx_0^2 + \tilde{h}^{-1}dx^2_{1,2} + \frac{3}{2}(r^2+l^2)ds^2_{\rm cycle} + \\ &\qquad + U^{-1}dr^2 + \frac{r^2}{4}\left[d\theta^2 + mB^2_{[1]}\right]\Big), \end{aligned} \tag{3.32}$$

$$\begin{aligned} e^{4\Phi/3} &= \tilde{h}^{-2/3}\tilde{H}^{-1}\,, \qquad B_{[2]} = -\frac{\Theta}{\tilde{H}\tilde{h}}\, dx^1\wedge dx^2\,, \\ C_{[1]} &= -Uf^{-1}\cos\theta\, B_{[1]}\,, \qquad C_{[3]} = -\frac{\Theta}{\tilde{H}\tilde{h}}Uf^{-1}\cos\theta\;\, dx^1\wedge dx^2\wedge B_{[1]}\,. \end{aligned} \tag{3.33}$$

Instead, if we reduced along the ∂_ψ isometry, we would break all the supersymmetry.

On the other hand, when reducing to 8d gauged sugra, we find a big difference between the commutative (3.1) and the NC (3.29) cases, so we analyse them separately. As discussed in section 2.3, to see if supersymmetry will be preserved in the SU(2) compactification, we have to transform the spinors to the SU(2) left-invariant w-base (2.37). To do so, we need to apply the rotation (2.38) to the Killing spinors.

If we are in the commutative case, it is easy to see that the corresponding spinors (3.3) become constant, independent of all the S^3 angles. Therefore, the compactification can be performed preserving all four supersymmetries. This is what allowed the authors of [17] to find such solution using 8d supergravity.

On the other hand, in the NC case, it can be checked that not even the metric can be put in a form that satisfies the reduction ansatz, so the compactification is simply not possible. As a consequence, the NC wrapped D6 solution (3.29) could have never been found with the usual gauged supergravity method.

4. Conclusions

We have constructed a supersymmetric configuration of D6-branes wrapping a Kähler four-cycle, with a non-vanishing background B-field (3.32). We have shown that it can be interpreted as a non-threshold bound state of D6-D4 branes, with the D4 dissolved in the worldvolume of the D6, and wrapped around the cycle. The problem has been analysed in 11d because, as we have shown, its compactification to 8d gauged sugra would have destroyed all the supersymmetries. Similarly, the reduction to type IIA along a U(1) isometry required some care to avoid this phenomenon. We note that the correct reduction produces a background where the fields depend on two transverse coordinates and it has therefore cohomogeneity two.

Despite the fact that it preserves four supercharges, the resulting metric is not Ricci-flat, so it does not have reduced holonomy. This is the usual situation for compactifications with background fluxes and, indeed, it would be nice to show that the four-cycle is calibrated in the sense of generalised calibrations [56].

JHEP10(2002)016

This configuration is expected to be dual in the IR of an $\mathcal{N}=2$ NC gauge theory in $2+1$ dimensions, with non commutativity along the spatial directions. Such field theory is non local in space, but local in time and it does not suffer from unitarity or causality problems. This solution will hopefully allow the study of non-perturbative properties of this NC field theory.

Acknowledgments

We would like to thank Toine Van Proeyen and Daniela Zanon for collaboration at earlier stages of this paper. We are also grateful to Roberto Casalbuoni, Jerome Gauntlett, Jaume Gomis, Giorgo Longhi, David Mateos, Carlos Núñez, Tomas Ortín, Alfonso Ramallo, Joan Simón and Paul Townsend for useful discussions. J.B. is supported by a grant from Ministerio de Ciencia y Tecnología. This work is partially supported by MCYT FPA, 2001-3598, and CIRIT, GC 2001SGR-00065. T.M. is supported by a grant from the Commissionat per a la Recerca de la Generalitat de Catalunya.

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[4] J.M. Maldacena and C. Núñez, *Supergravity description of field theories on curved manifolds and a no go theorem*, *Int. J. Mod. Phys.* **A 16** (2001) 822 [hep-th/0007018].

[5] J.M. Maldacena and C. Núñez, *Towards the large-N limit of pure $N=1$ super Yang-Mills*, *Phys. Rev. Lett.* **86** (2001) 588 [hep-th/0008001].

[6] B.S. Acharya, J.P. Gauntlett and N. Kim, *Fivebranes wrapped on associative three-cycles*, *Phys. Rev.* **D 63** (2001) 106003 [hep-th/0011190].

[7] H. Nieder and Y. Oz, *Supergravity and D-branes wrapping special lagrangian cycles*, *J. High Energy Phys.* **03** (2001) 008 [hep-th/0011288].

[8] J.P. Gauntlett, N. Kim and D. Waldram, *M-fivebranes wrapped on supersymmetric cycles*, *Phys. Rev.* **D 63** (2001) 126001 [hep-th/0012195].

[9] C. Núñez, I.Y. Park, M. Schvellinger and T.A. Tran, *Supergravity duals of gauge theories from F(4) gauged supergravity in six dimensions*, *J. High Energy Phys.* **04** (2001) 025 [hep-th/0103080].

[10] J.D. Edelstein and C. Núñez, *D6 branes and M-theory geometrical transitions from gauged supergravity*, *J. High Energy Phys.* **04** (2001) 028 [hep-th/0103167].

[11] M. Schvellinger and T.A. Tran, *Supergravity duals of gauge field theories from* $\mathrm{SU}(2)\times\mathrm{U}(1)$ *gauged supergravity in five dimensions*, *J. High Energy Phys.* **06** (2001) 025 [hep-th/0105019].

[12] J.M. Maldacena and H. Nastase, *The supergravity dual of a theory with dynamical supersymmetry breaking*, *J. High Energy Phys.* **09** (2001) 024 [hep-th/0105049].

[13] J.P. Gauntlett, N. Kim, S. Pakis and D. Waldram, *Membranes wrapped on holomorphic curves*, *Phys. Rev.* **D 65** (2002) 026003 [hep-th/0105250].

[14] R. Hernandez, *Branes wrapped on coassociative cycles*, *Phys. Lett.* **B 521** (2001) 371 [hep-th/0106055].

[15] J.P. Gauntlett, N. Kim, D. Martelli and D. Waldram, *Wrapped fivebranes and $N = 2$ super Yang-Mills theory*, *Phys. Rev.* **D 64** (2001) 106008 [hep-th/0106117].

[16] F. Bigazzi, A.L. Cotrone and A. Zaffaroni, *$N = 2$ gauge theories from wrapped five-branes*, *Phys. Lett.* **B 519** (2001) 269 [hep-th/0106160].

[17] J. Gomis and T. Mateos, *D6-branes wrapping Kähler four-cycles*, *Phys. Lett.* **B 524** (2002) 170 [hep-th/0108080].

[18] J.P. Gauntlett and N. Kim, *M-fivebranes wrapped on supersymmetric cycles, 2*, *Phys. Rev.* **D 65** (2002) 086003 [hep-th/0109039].

[19] J. Gomis and J.G. Russo, *$D = 2 + 1$ $N = 2$ Yang-Mills theory from wrapped branes*, *J. High Energy Phys.* **10** (2001) 028 [hep-th/0109177].

[20] J.P. Gauntlett, N.-W. Kim, D. Martelli and D. Waldram, *Fivebranes wrapped on slag three-cycles and related geometry*, *J. High Energy Phys.* **11** (2001) 018 [hep-th/0110034].

[21] J. Gomis, *On SUSY breaking and χSB from string duals*, *Nucl. Phys.* **B 624** (2002) 181 [hep-th/0111060].

[22] G. Curio, B. Körs and D. Lüst, *Fluxes and branes in type-II vacua and M-theory geometry with G_2 and* Spin(7) *holonomy*, *Nucl. Phys.* **B 636** (2002) 197 [hep-th/0111165].

[23] P. Di Vecchia, H. Enger, E. Imeroni and E. Lozano-Tellechea, *Gauge theories from wrapped and fractional branes*, *Nucl. Phys.* **B 631** (2002) 95 [hep-th/0112126].

[24] R. Hernandez and K. Sfetsos, *An eight-dimensional approach to G_2 manifolds*, *Phys. Lett.* **B 536** (2002) 294 [hep-th/0202135].

[25] J.P. Gauntlett, N. Kim, S. Pakis and D. Waldram, *M-theory solutions with AdS factors*, *Class. and Quant. Grav.* **19** (2002) 3927 [hep-th/0202184].

[26] U. Gürsoy, C. Núñez and M. Schvellinger, *RG flows from* Spin(7)*, CY 4-fold and HK manifolds to AdS, Penrose limits and PP waves*, *J. High Energy Phys.* **06** (2002) 015 [hep-th/0203124].

[27] R. Hernandez and K. Sfetsos, *Branes with fluxes wrapped on spheres*, *J. High Energy Phys.* **07** (2002) 045 [hep-th/0205099].

[28] P. Di Vecchia, A. Lerda and P. Merlatti, *$N = 1$ and $N = 2$ super Yang-Mills theories from wrapped branes*, hep-th/0205204.

[29] R. Apreda, F. Bigazzi, A.L. Cotrone, M. Petrini and A. Zaffaroni, *Some comments on $N = 1$ gauge theories from wrapped branes*, *Phys. Lett.* **B 536** (2002) 161 [hep-th/0112236].

[30] M. Naka, *Various wrapped branes from gauged supergravities*, hep-th/0206141.

[31] M. Bershadsky, C. Vafa and V. Sadov, *D-branes and topological field theories*, *Nucl. Phys.* **B 463** (1996) 420 [hep-th/9511222].

[32] A. Hashimoto and N. Itzhaki, *Non-commutative Yang-Mills and the AdS/CFT correspondence*, *Phys. Lett.* **B 465** (1999) 142 [hep-th/9907166].

[33] J.M. Maldacena and J.G. Russo, *Large-N limit of non-commutative gauge theories*, *J. High Energy Phys.* **09** (1999) 025 [hep-th/9908134].

[34] M. Alishahiha, Y. Oz and M.M. Sheikh-Jabbari, *Supergravity and large-N noncommutative field theories*, *J. High Energy Phys.* **11** (1999) 007 [hep-th/9909215].

[35] D.S. Berman et al., *Holographic noncommutativity*, *J. High Energy Phys.* **05** (2001) 002 [hep-th/0011282].

[36] H. Larsson, *A note on half-supersymmetric bound states in M-theory and type-IIA*, *Class. and Quant. Grav.* **19** (2002) 2689 [hep-th/0105083].

[37] A. Salam and E. Sezgin, $D = 8$ *supergravity*, *Nucl. Phys.* **B 258** (1985) 284.

[38] M.J. Duff, H. Lu and C.N. Pope, *Supersymmetry without supersymmetry*, *Phys. Lett.* **B 409** (1997) 136 [hep-th/9704186].

[39] E. Cremmer, B. Julia and J. Scherk, *Supergravity theory in 11 dimensions*, *Phys. Lett.* **B 76** (1978) 409.

[40] J. Gomis and T. Mehen, *Space-time noncommutative field theories and unitarity*, *Nucl. Phys.* **B 591** (2000) 265 [hep-th/0005129].

[41] N. Seiberg, L. Susskind and N. Toumbas, *Space/time non-commutativity and causality*, *J. High Energy Phys.* **06** (2000) 044 [hep-th/0005015].

[42] P.K. Townsend, *The eleven-dimensional supermembrane revisited*, *Phys. Lett.* **B 350** (1995) 184 [hep-th/9501068].

[43] E. Witten, *String theory dynamics in various dimensions*, *Nucl. Phys.* **B 443** (1995) 85 [hep-th/9503124].

[44] G.W. Gibbons and S.W. Hawking, *Gravitational multi-instantons*, *Phys. Lett.* **B 78** (1978) 430.

[45] I.C.G. Campbell and P.C. West, $N = 2$ $D = 10$ *nonchiral supergravity and its spontaneous compactification*, *Nucl. Phys.* **B 243** (1984) 112.

[46] M. Huq and M.A. Namazie, *Kaluza-Klein supergravity in ten-dimensions*, *Class. and Quant. Grav.* **2** (1985) 293, erratum *ibid.* **2** (1985) 597.

[47] P.K. Townsend, *M-theory from its superalgebra*, hep-th/9712004.

[48] J.M. Camino and A.V. Ramallo, *M-theory giant gravitons with c field*, *Phys. Lett.* **B 525** (2002) 337 [hep-th/0110096].

[49] B.E.W. Nilsson and C.N. Pope, *Hopf fibration of eleven-dimensional supergravity*, *Class. and Quant. Grav.* **1** (1984) 499.

[50] T. Ortin, *A note on Lie-Lorentz derivatives*, *Class. and Quant. Grav.* **19** (2002) L143 [hep-th/0206159].

[51] E. Bergshoeff, R. Kallosh and T. Ortin, *Duality versus supersymmetry and compactification*, *Phys. Rev.* **D 51** (1995) 3009 [hep-th/9410230].

[52] J. Scherk and J.H. Schwarz, *How to get masses from extra dimensions*, *Nucl. Phys.* **B 153** (1979) 61.

[53] M. Cvetič, G.W. Gibbons, H. Lu and C.N. Pope, *Ricci-flat metrics, harmonic forms and brane resolutions*, hep-th/0012011.

[54] J. Gomis, *D-branes, holonomy and M-theory*, *Nucl. Phys.* **B 606** (2001) 3 [hep-th/0103115].

[55] H.J. Boonstra, K. Skenderis and P.K. Townsend, *The domain wall/QFT correspondence*, *J. High Energy Phys.* **01** (1999) 003 [hep-th/9807137].

[56] J. Gutowski, G. Papadopoulos and P.K. Townsend, *Supersymmetry and generalized calibrations*, *Phys. Rev.* **D 60** (1999) 106006 [hep-th/9905156].